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Analytical expressions for transient melting of polymer pellet sliding against adiabatic wall

K.L. Yung, Y. Xu*, K.H. Lau

Department of Manufacturing Engineering, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong, People's Republic of China

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Abstract

The melting of polymer caused by friction before the solid plug is formed is an important phenomenon in the plastic injection process. To analyze the melting process caused by solid particles sliding against the bellow, a method that can simulate behavior of each particle during the calculation should be used. Particle element numerical method is hence adopted in the analysis for this research to take into consideration the behavior of each particle, where the melting of pellets caused by friction against flight and screw are assumed as friction against adiabatic walls. In this paper, analytical expressions of the transient melting process for spherical polymer pellets sliding against adiabatic wall are derived, which is essential for the numerical simulation of melting process in solid conveying section by particle element method.

Keywords: Adiabatic; Friction; Melting

1. Introduction

In the analysis of the plastic injection process inside the injector, experiments [1,2] have shown that, in the solid conveying section, melting occurs before the solid plug is formed. Heat generated by friction between pellets and wall cause melting at the interface of pellets and wall. To include this melting phenomenon into the numerical particle element analysis, an analytical mathematical expression for the transient melting process is necessary.

Until now, studies on close-contact melting processes are mostly on steady state [3–6]. Very few unsteady analyses have been reported. Hong and Saito [7] appears to be the first to investigate the transient process as a separate subject. Using a sophisticated model, they numerically solved full governing equations for unsteady two-dimensional flow and heat transfer during close-contact melting between ice kept at its freezing temperature and an isothermally heated flat plate. The same method has also been applied to the case of constant wall heat flux [8]. Yoo [9] presented an analytical solution to the unsteady close-contact melting with two kinds of boundary conditions: constant wall temperature and constant wall heat flux. The initial solid temperature was assumed at melting point. None of these transient models included the heat generated by friction into the equation.

To study the solid conveying process in plastics extrusion, particle element numerical method is a good option which enables us to analyze behavior of each solid particle during the process and take into account parameters of each solid particle during the calculation. The present study is intended to derive a set of analytical solution for a simplified model of friction-induced unsteady contact melting. In this model, the geometry of polymer pellet is taken to be spherical. The contact surface is assumed to maintain the shape of a circular plate during melting (with radius R). This plate is initially caused by elastic deformation under the normal force F_N (see Fig. 1). The initial temperature at the interface of polymer and wall is assumed to be at the melting point T_m. The initial temperature of solid polymer is T_{s0}. Boundary condition at the wall is adiabatic.

2. Modeling

The physical system considered in this work is depicted schematically in Fig. 1 together with the coordinate. An adiabatic plate moves at a constant relative velocity U in x direction due to the tangential force F_T. An external normal force F_N is applied on the sphere to keep the contact between the polymer sphere and the wall. At $t = 0$, temperature at contact interface is T_m. The temperature of solid at

* Corresponding author.
E-mail address: drxuyan@bigfoot.com (Y. Xu).

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Nomenclature

A	area of contact surface
c	polymer specific heat
F_N	normal force
$\tilde{F}_N$	dimensionless normal force, $F_N/(\alpha\mu)$
F_T	tangential force
h	liquid film thickness
$\tilde{h}$	dimensionless liquid film thickness h/R
$\hat{h}$	normalized liquid film thickness $\tilde{h}/\tilde{h}_C$
K	polymer thermal conductivity
P	pressure in the liquid film
Pr	Prandtl number, $\mu c/K$
Q	volume flow rate
R	radius of circular contact area
t	time
$\tilde{t}$	dimensionless time, $t\alpha/R^2$
$\hat{t}$	normalized time, $\tilde{t}\tilde{V}_C/\tilde{h}_C$
T	temperature
T_m	melting temperature
T_{s0}	solid temperature
V_x, V_y, V_z	velocity components (Fig. 1)
U	velocity of relative motion
$\tilde{U}$	dimensionless relative velocity, UR/α
V	solid descending velocity
$\tilde{V}$	dimensionless descending velocity, VR/α
$\hat{V}$	normalized descending velocity, $\tilde{V}/\tilde{V}_C$
x, y, z	Cartesian coordinates (Fig. 1)
ΔT	$T_m - T_{s0}$
Ste	Stefan number, $c\,\Delta T/\lambda$
α	liquid thermal diffusivity, $K/(\rho c)$
λ	latent heat of fusion
$\tilde{\lambda}$	dimensionless latent heat of fusion, $\lambda R^2/\alpha^2$
μ	viscosity
ρ	density

Subscripts

C	steady state
x, y	x, y direction

$y \rightarrow \infty$ is maintained at T_{s0}. This assumption is based on the facts that the diffusivity coefficient of polymer is low and the size of the pellet is much larger than film thickness. Due to the heat generated by viscous dissipation, the polymer starts to melt. As the melting proceeds, part of the liquid generated along the phase change front fills up the growing gap between polymer solid sphere and plate, while the rest is forced to flow and to be squeezed out through the peripheral openings around the contact area by the solid descending motion. Both the liquid film thickness h and the solid descending velocity V vary with time, and eventually attain the quasi-steady state where the melting rate coincides with the solid descending velocity.

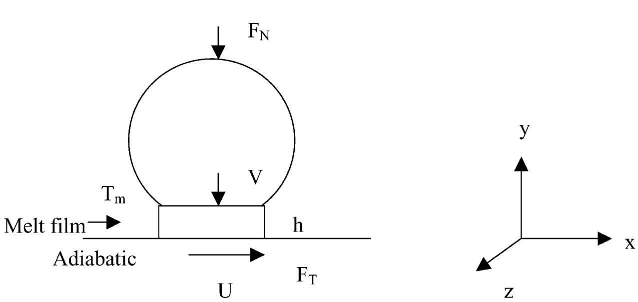

Fig. 1. Physical system.

In order to approach the problem analytically, simplifications have been introduced within the extent of preserving the fundamental features of frictional melting. (1) The solid to be melted is assumed to be a circular semi-infinite bar with radius R. (2) The thin film approximation holds for the liquid layer throughout the whole process. (3) At $t = 0$, temperature at contact area is at melting temperature T_m, while the temperature of polymer sphere at $y \rightarrow \infty$ is T_{s0}. (4) Convective heat transfer is negligible. (5) The pressure in the film is uniform in cross-wise direction [10]. (6) Assume the densities of solid and melt are the same, so that there is no volume change during phase change.

Based on previous works [4,9,11], the time-dependent energy balance at the solid–liquid interface is expressed as

$$-k\left(\frac{\partial T}{\partial y}\right)_{y=h} = \rho(\lambda + c\,\Delta T)\left(V + \frac{\mathrm{d}h}{\mathrm{d}t}\right) \tag{1}$$

where $\Delta T = T_m - T_{s0}$, h the melt film thickness, k the coefficient of thermal conductivity, ρ the density, c the specific heat, λ the latent heat of fusion and V the solid descending velocity.

Normal force balance:

$$F_N = \int\int_A P(x,z)\mathrm{d}A \tag{2}$$

where A is the area of contact surface and P is the pressure in melt film.

Neglecting shear in other directions since movement U is dominant, the total shear force of liquid film is expressed as

$$F_T = -\int\int_A \mu\frac{\partial V_x}{\partial y}\bigg|_{y=0}\,\mathrm{d}A \approx -\frac{\mu U \pi R^2}{h} \tag{3}$$

where μ is the viscosity of melt, U the velocity of relative motion and V_x the moving velocity in x direction (Fig. 1).

The continuity equation is

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0 \tag{4}$$

The momentum equations are

$$\frac{\partial P}{\partial x} = \mu\frac{\partial^2 V_x}{\partial y^2} \tag{5}$$

$$\frac{\partial P}{\partial z} = \mu\frac{\partial^2 V_z}{\partial y^2} \tag{6}$$

Since the shear in x direction is dominant, conductions in x

and z directions and convections are negligible under assumptions (2) and (4). The energy equation can be expressed as

$$\frac{\partial^2 T}{\partial y^2} = -\frac{\mu}{K}\left(\frac{\partial V_x}{\partial y}\right)^2 = -\frac{\mu}{K}\left(\frac{U}{h}\right)^2 \quad (7)$$

where U is the velocity of contact plate in x direction.

Substituting boundary condition $(\partial T/\partial y)|_{y=0} = 0$ (adiabatic wall) into Eq. (7) yields

$$\frac{\partial T}{\partial y} = -\frac{\mu}{K}\left(\frac{U}{h}\right)^2 y \quad (8)$$

so

$$\left.\frac{\partial T}{\partial y}\right|_{y=h} = -\frac{\mu}{K}\frac{U^2}{h} \quad (9)$$

Combining Eqs. (1) and (9) yields

$$\mu\frac{U^2}{h} = \rho(\lambda + c\,\Delta T)\left(V + \frac{\mathrm{d}h}{\mathrm{d}t}\right) \quad (10)$$

Since, for steady state $\mathrm{d}h/\mathrm{d}t = 0$, so steady state film thickness is

$$h_c = \frac{\mu U^2}{\rho(\lambda + c\,\Delta T)V} \quad (11)$$

Solving Eqs. (5) and (6) with boundary conditions: $V_x = U$, $V_y = V_z = 0$ at $y = 0$ and $V_x = V_z = 0$, $V_y = -V$ at $y = h$, we obtain

$$V_x(x,y,z) = \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right)y(y-h) + U\left(1 - \frac{y}{h}\right) \quad (12)$$

$$V_z(x,y,z) = \frac{1}{2\mu}\left(\frac{\partial P}{\partial z}\right)y(y-h) \quad (13)$$

Flow rates in x and z directions:

$$Q_x = \int_0^h V_x\,\mathrm{d}y = \frac{1}{12\mu}\left(-\frac{\partial P}{\partial x}\right)h^3 + \frac{1}{2}Uh \quad (14)$$

$$Q_z = \int_0^h V_z\,\mathrm{d}y = \frac{1}{12\mu}\left(-\frac{\partial P}{\partial z}\right)h^3 \quad (15)$$

Substitute Eqs. (14) and (15) into continuity equation and integrated with respect to y, yields

$$\frac{\partial Q_x}{\partial x} - V + \frac{\partial Q_z}{\partial z} = 0 \quad (16)$$

It results in the following equation for $P(x,z)$:

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial z^2} = -\frac{12\mu V}{h^3} \quad (17)$$

Note that right-hand side of Eq. (17) is independent of x and z.

Since the geometry of contact area is circular, Eq. (17) can be expressed in cylindrical coordinate:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{1}{r}\frac{\partial P}{\partial \theta}\right) = -\frac{12\mu V}{h^3} \quad (18)$$

For simplicity suppose P is independent of θ, then:

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = -\frac{12\mu V}{h^3} \quad (19)$$

With boundary condition $P = 0$ at $r = R$, solution of Eq. (19) is

$$P = \frac{3\mu V}{h^3}(R^2 - r^2) \quad (20)$$

Hence, normal force as derived from Eq. (2) is

$$F_{\mathrm{N}} = \iint_A \frac{3\mu V}{h^3}(R^2 - r^2)\mathrm{d}A = \int_0^{2\pi}\int_0^R r\frac{3\mu V}{h^3}(R^2 - r^2)\mathrm{d}r\,\mathrm{d}\theta = \frac{3}{2}\frac{\mu V R^4 \pi}{h^3} \quad (21)$$

Combining Eqs. (10) and (21) yields

$$\frac{\mathrm{d}h}{\mathrm{d}t} - \frac{\mu U^2}{\rho(\lambda + c\,\Delta T)}\frac{1}{h(t)} + \frac{2h^3(t)F_{\mathrm{N}}}{3\mu R^4 \pi} = 0 \quad (22)$$

with initial condition $h(0) = 0$, film thickness $h(t)$ can be

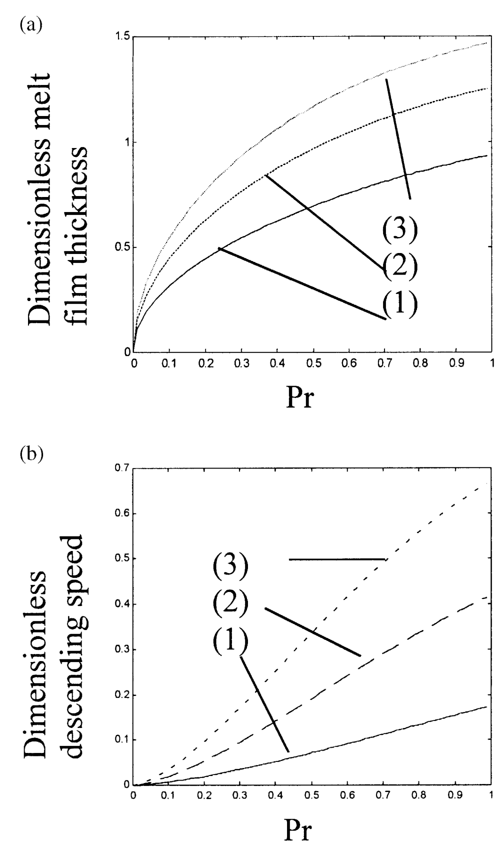

Fig. 2. The effect of Prandtl number on melt film thickness (a) and descending speed (b). (1) $Pr = 1$; (2) $Pr = 2$; and (3) $Pr = 3$.

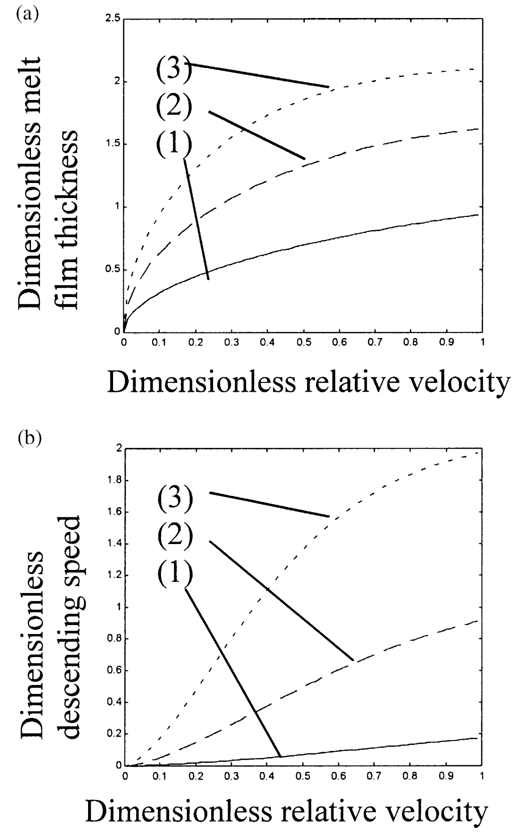

Fig. 3. The effect of relative velocity on melt films thickness (a) and descending speed (b). (1) $\tilde{U} = 1$; (2) $\tilde{U} = 2$; and (3) $\tilde{U} = 3$.

solved from Eq. (22):

$$h(t) = (B/C)^{1/4}\sqrt{\frac{e^{4t\sqrt{BC}} - 1}{e^{4t\sqrt{BC}} + 1}} \tag{23}$$

Solid descending velocity is derived from Eq. (21):

$$V(t) = \frac{2F_N}{3\mu R^4 \pi}(B/C)^{3/4}\left(\frac{e^{4t\sqrt{BC}} - 1}{e^{4t\sqrt{BC}} + 1}\right)^{3/2} \tag{24}$$

where $B = (\mu U^2)/(\rho(\lambda + c\,\Delta T))$, $C = (2F_N)/(3\mu R^4 \pi)$.

3. Nondimensionalization and normalization

In order to verify the characteristic parameters pertinent to the present system, the model equations are nondimensionalized. Let the characteristic length be R.

Dimensionless normal force: $\tilde{F}_N = F_N/(\alpha\mu)$
Dimensionless liquid film thickness: $\tilde{h} = h/R$
Prandtl number: $Pr = \mu c/K$
Dimensionless time: $\tilde{t} = t\alpha/R^2$
Dimensionless relative velocity: $\tilde{U} = UR/\alpha$
Dimensionless descending velocity: $\tilde{V} = VR/\alpha$
Stefan number: $Ste = c\,\Delta T/\lambda$
Dimensionless latent heat of fusion: $\tilde{\lambda} = \lambda R^2/\alpha^2$

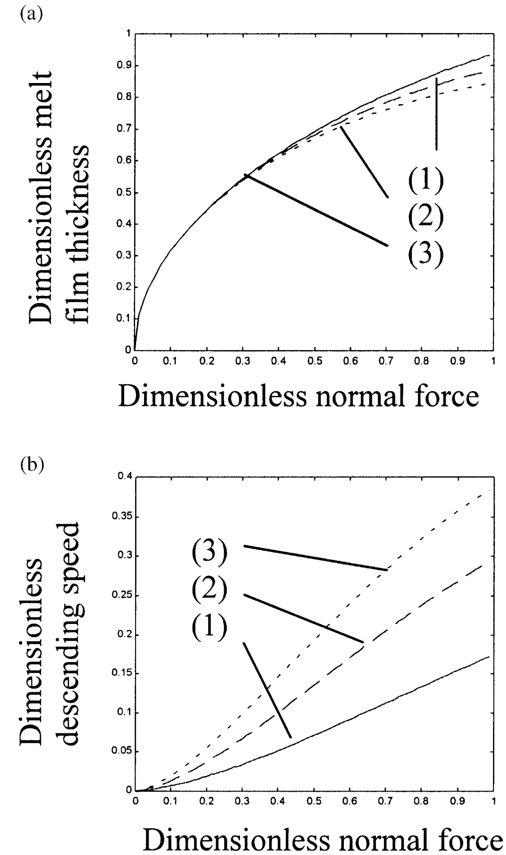

Fig. 4. The effect of normal force on melt film thickness (a) and descending speed (b). (1) $\tilde{F}_N = 1$; (2) $\tilde{F}_N = 2$; and (3) $\tilde{F}_N = 3$.

Eq. (10) can be rewritten in terms of these dimensionless quantities as

$$\frac{Pr\tilde{U}^2}{\tilde{h}} = \tilde{\lambda}(1 + Ste)\left(\tilde{V} + \frac{d\tilde{h}}{d\tilde{t}}\right) \tag{25}$$

Eq. (21) can be rewritten as

$$\tilde{F}_N = \frac{3\pi\tilde{V}}{2\tilde{h}^3} \tag{26}$$

Combining Eqs. (25) and (26), we obtain:

$$\frac{d\tilde{h}}{d\tilde{t}} - \frac{Pr\tilde{U}^2}{\tilde{\lambda}(1 + Ste)}\frac{1}{\tilde{h}(\tilde{t})} + \frac{2\tilde{h}^3(\tilde{t})\tilde{F}_N}{3\pi} = 0 \tag{27}$$

with initial condition $\tilde{h}(0) = 0$, we get

$$\tilde{h}(\tilde{t}) = (B/C)^{1/4}\sqrt{\frac{e^{4\tilde{t}\sqrt{BC}} - 1}{e^{4\tilde{t}\sqrt{BC}} + 1}} \tag{28}$$

$$\tilde{V}(\tilde{t}) = \frac{2\tilde{F}_N}{3\pi}(B/C)^{3/4}\left(\frac{e^{4t\sqrt{BC}} - 1}{e^{4t\sqrt{BC}} + 1}\right)^{3/2} \tag{29}$$

where $B = (Pr\tilde{U}^2)/(\tilde{\lambda}(1 + Ste))$, $C = (2\tilde{F}_N)/(3\pi)$.

The steady state ($t \to \infty$) dimensionless film thickness

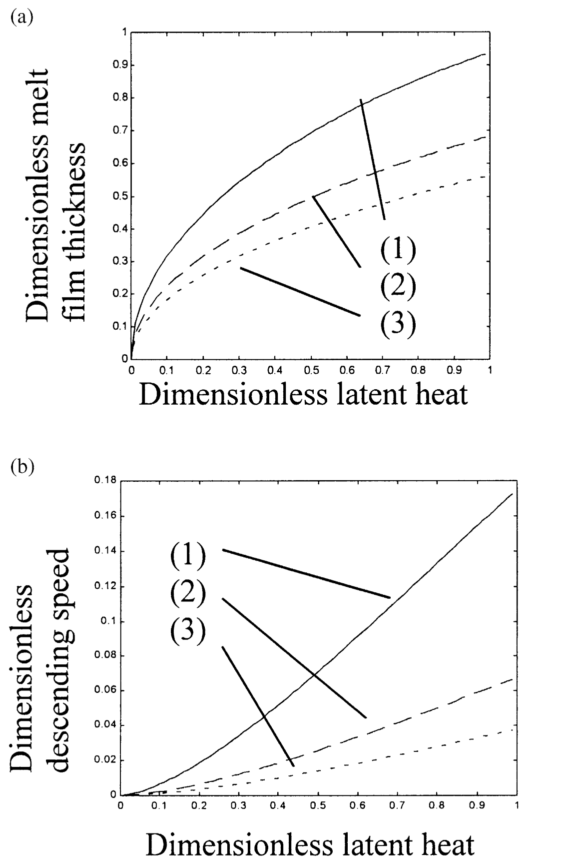

Fig. 5. The effect of latent heat of fusion on melt film thickness (a) and descending speed (b). (1) $\tilde{\lambda} = 1$; (2) $\tilde{\lambda} = 2$; and (3) $\tilde{\lambda} = 3$.

and solid descending velocity are

$$\tilde{h}_C = \left(\frac{3\pi Pr \tilde{U}^2}{2\tilde{F}_N \tilde{\lambda}(1 + Ste)} \right)^{1/4} \tag{30}$$

$$\tilde{V}_C = \frac{2\tilde{F}_N}{3\pi} \left(\frac{3\pi Pr \tilde{U}^2}{2\tilde{F}_N \tilde{\lambda}(1 + Ste)} \right)^{3/4} \tag{31}$$

Normalization in reference to the steady solution is helpful to a generalized analysis of transient process. According to the definition of normalized variables, Eqs. (25) and (26) are reduced to

$$\hat{V} + \frac{d\hat{h}}{d\hat{t}} = \frac{1}{\hat{h}} \tag{32}$$

$$\hat{V} = \hat{h}^3 \tag{33}$$

Combining Eqs. (32) and (33), we obtain

$$\frac{d\hat{h}}{d\hat{t}} - \frac{1}{\hat{h}(\hat{t})} + \hat{h}^3(\hat{t}) = 0 \tag{34}$$

Solving Eq. (34) along with the initial condition $\hat{h}(0) = 0$,

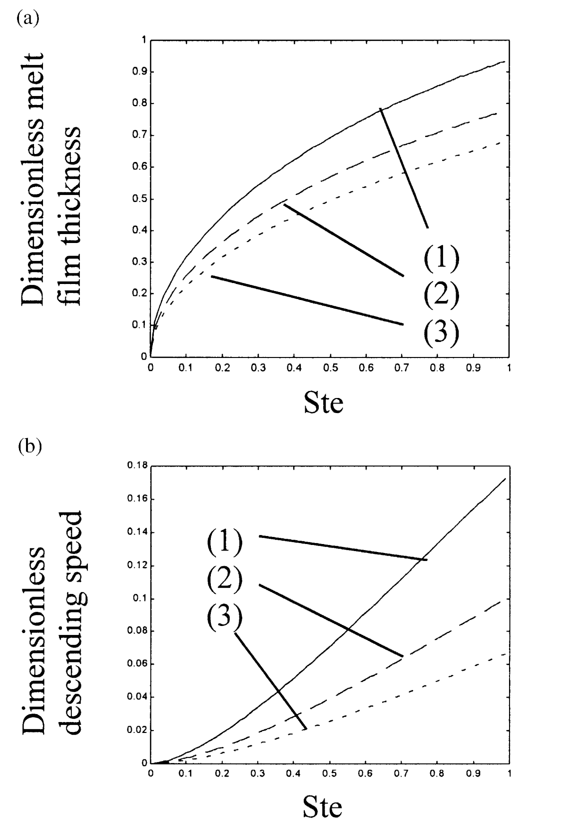

Fig. 6. The effect on Stefan number relative to solid on melt film thickness (a) and descending speed (b). (1) *Ste* = 1; (2) *Ste* = 2; and (3) *Ste* = 3.

results in:

$$\hat{h}(\hat{t}) = \left(\frac{e^{4\hat{t}} - 1}{e^{4\hat{t}} + 1} \right)^{1/2} \tag{35}$$

$$\hat{V}(\hat{t}) = \left(\frac{e^{4\hat{t}} - 1}{e^{4\hat{t}} + 1} \right)^{3/2} \tag{36}$$

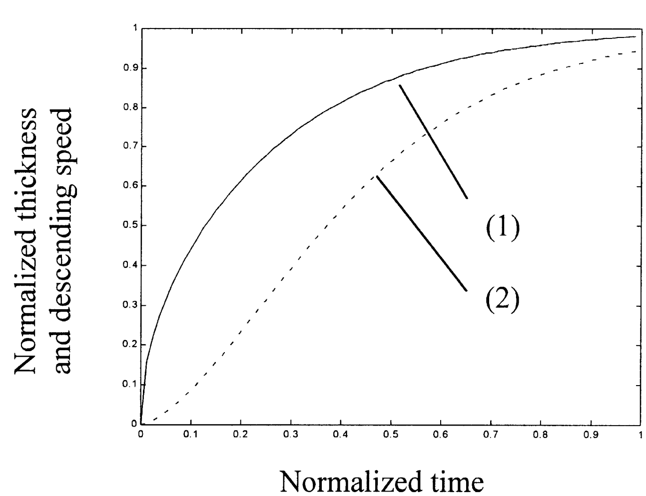

Fig. 7. Relationship between normalized thickness and descending speed with normalized time. (1) $\hat{h}(\hat{t})$ and (2) $\hat{V}(\hat{t})$.

4. Results and discussion

Expressions for the transient melting of polymer due to friction against the wall has been derived. The effects of the normal force, the difference between solid initial temperature and melting point temperature, latent heat of fusion, Prandtl number, velocity of relative motion on time-dependent melt film thickness and melting rate (solid descending speed) are all clearly reflected in Figs. 2–6, and the analytical expressions Eqs. (28) and (29). The transient features of melting film thickness and melting rate (solid descending speed) are shown in Fig. 7. It is shown that higher values in Prandtl number, temperature of solid polymer and velocity of relative motion, lower values in normal force and latent heat of fusion will cause higher melting rate and thicker melting film. Higher normal force causes higher melting rate and thinner melt film. However, the effect of normal force on melt film thickness is much less than other factors.

5. Conclusions

This paper presents a set of analytical expressions for transient melting process caused by viscous dissipation. The model derived here can be utilized to describe the melting process when polymer pellets are sliding against an adiabatic wall. By including this model into particle element numerical simulations, the movement and melting of solid pellets during extrusion become clear. Hence, this model is very useful for analyzing the solid conveying process in plastics extrusion and optimizing the design of the entire injection-molding machine. This model can also be used in other applications related to melting caused by friction (especially autothermal extrusion [12,13], etc.).

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